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# Relational algebra operations

| Operation | Notation | Definition | Schema Description | Degree | Cardinality (c) |
|---|---|---|---|---|---|
| Selection | $\sigma_\theta(\mathbf{r})$ | | $R$ | $n$ | $0 \leq c \leq N$ |
| | Returns tuples of $r$ satisfying the predicate $\theta$. | | | | |
| Projection | $\pi_{\mathbf{A_1,A_2}\ldots}(\mathbf{r})$ | | $\{A_1, A_2 \ldots\}$ | $\lvert\{A_1, A_2 \ldots\}\rvert$ | $1 \leq c \leq N$ |
| | Returns tuples of $r$ consisting only of attributes $A_1, A_2 \ldots$ (eliminates duplicates). | | | | |
| Union[1] | $\mathbf{r} \cup \mathbf{s}$ | | $R$ | $n, m$ | $\max\{N, M\} \leq c \leq N + M$ |
| | Returns all tuples of $r$ and all of $s$ (eliminates duplicates). | | | | |
| Difference[1] | $\mathbf{r} - \mathbf{s}$, $\mathbf{r} \backslash \mathbf{s}$ | | $R$ | $n, m$ | $\max\{0, N - M\} \leq c \leq N$ |
| | Returns all tuples of $r$ that are not in $s$. | | | | |
| Cartesian product | $\mathbf{r} \times \mathbf{s}$ | | $R.S$ | $n + m$ | $NM$ |
| | Concatenates each tuple of $r$ with each tuple of $s$. | | | | |
| Intersection[1] | $\mathbf{r} \cap \mathbf{s}$ | r-(r-s) | $R$ | $n, m$ | $0 \leq c \leq \min\{N, M\}$ |
| | Returns all tuples of $r$ that are also in $s$. | | | | |
| Theta join | $\mathbf{r} \bowtie_\theta \mathbf{s}$ | $\sigma_\theta(r \times s)$ | $R.S$ | $n + m$ | $0 \leq c \leq NM$ |
| | Returns tuples of cartesian product of $r$ and $s$ satisfying the predicate $\theta$. | | | | |
| Equijoin | $\mathbf{r} \bowtie_{\theta_=} \mathbf{s}$ | $\sigma_{\theta_=}(r \times s)$ | $R.S$ | $n + m$ | $0 \leq c \leq NM$ |
| | Returns tuples of cartesian product of $r$ and $s$ satisfying the predicate $\theta_=$. | | | | |
| Natural join[2] | $\mathbf{r} \bowtie \mathbf{s}$ | $\pi_{R \cup S}(\sigma_{r.A_1 = s.A_1 \wedge \ldots}(r \times s))$ | $R \cup S$ | $n + m - cnm$ | $0 \leq c \leq NM$ |
| | Returns equijoin of $r$ and $s$ over all common attributes where one occurrence of each common attribute is eliminated. | | | | |
| Left outer join | $\mathbf{r} \rtimes \mathbf{s}$ | / | $R \cup S$ | $n + m - cnm$ | $N \leq c \leq NM$ |
| | Returns natural join of $r$ and $s$ where tuples of $r$ having no matching values in common attributes of $s$ are also included in the result[3]. Comment: $\pi_R(r \rtimes s) = r$. | | | | |
| Right outer join | $\mathbf{r} \ltimes \mathbf{s}$ | / | $R \cup S$ | $n + m - cnm$ | $M \leq c \leq NM$ |
| | Returns natural join of $r$ and $s$ where tuples of $s$ having no matching values in common attributes of $r$ are also included in the result[3]. Comment: $\pi_S(r \ltimes s) = s$. | | | | |
| Full outer join | $\mathbf{r} \times \mathbf{s}$ | / | $R \cup S$ | $n + m - cnm$ | $N + M \leq c \leq NM$ |
| | Returns natural join of $r$ and $s$ where tuples of $r$ or $s$ having no matching values in common attributes of the other relation are also included in the result[3]. | | | | |
| Semijoin | $\mathbf{r} \triangleright_\theta \mathbf{s}$ | $\pi_R(\sigma_\theta(r \times s))$ | $R$ | $n$ | $0 \leq c \leq N$ |
| | Returns tuples of $r$ that participate in theta join of $r$ and $s$ (due to predicate $\theta$). | | | | |
| Division[4] | $\mathbf{r}/\mathbf{s}$, $\mathbf{r} \div \mathbf{s}$ | $\pi_{R-S}(r) -$<br>$- \pi_{R-S}((\pi_{R-S}(r) \times s) - r)$ | $R - S$ | $n - m$ | $0 \leq c \leq N/M$ |
| | Returns tuples of $r$ consisting only of attributes in $R - S$ (denoted $x$), thus that for *every* tuple in $s$ (denoted $y$) there exists a tuple in $r$ equal to concatenation of $x$ and $y$. | | | | |
| Aggregate | $\tau_{\mathbf{AL}}(\mathbf{r})$ | / | / | $\lvert AL\rvert$ | 1 |
| | Applies aggregate function list $AL$ to the relation $r$. | | | | |
| Grouping | $_{\mathbf{GA}}\tau_{\mathbf{AL}}(\mathbf{r})$ | / | / | $\lvert GA\rvert + \lvert AL\rvert$ | $1 \leq c \leq \prod_{A_i \in GA} \lvert dom(A_i)\rvert$ |
| | Groups tuples of $r$ by grouping attributes $GA$ and then applies aggregate function list $AL$ to these groups. | | | | |
| Rename | $\rho_{\mathbf{r}(\mathbf{A_1,A_2}\ldots)}(\mathbf{s})$ | / | $\{A_1, A_2 \ldots\}$ | $\lvert\{A_1, A_2 \ldots\}\rvert$ | $M$ |
| | Renames the relation $s$ and all of its attributes (new names are $r$ and $A_1, A_2 \ldots$). | | | | |
| Assignment | $\mathbf{r}(\mathbf{A_1}, \mathbf{A_2} \ldots) \leftarrow \mathbf{s}$ | / | $\{A_1, A_2 \ldots\}$ | $\lvert\{A_1, A_2 \ldots\}\rvert$ | $M$ |
| | Assigns name $r$ to relation $s$ and $A_1, A_2 \ldots$ to its attributes. | | | | |

$r, s$ → relations, with relation schemas $R, S$ ($Sh(r) = R$, $Sh(s) = S$)
$n, m$ → degrees of the relations ($deg(r) = n$, $deg(s) = m$)
$cnm$ → number of common attributes in the relations $r$ and $s$
$N, M$ → cardinalities of the relations ($card(r) = N$, $card(s) = M$)
$A_i$ → attribute
$\theta$ → logical predicate
$\theta_=$ → logical predicate which contains only equality comparisons and logical conjunction $\wedge$
$AL$ → list of aggregate functions[5] and corresponding attributes (e.g. "*COUNT* $A_1$, *SUM* $A_2 \ldots$")
$GA$ → grouping attributes (e.g. "$A_1, A_2 \ldots$"). They must differ from those in $AL$

[0] Author takes no responsibility for the errors in the text.
[1] $r$ and $s$ must be union-compatible.
[2] When $r$ and $s$ have no common attributes, natural join equals cartesian product.
[3] Unknown values are set to *NULL*.
[4] Attributes of $s$ must be a subset of attributes of $r$, $S \subseteq R$.
[5] Can be one of *COUNT*, *SUM*, *AVG*, *MAX* or *MIN*.