Digital Signal Processing - Laboratory sessions

3rd Laboratory Session

1. **Sampling**

   Sampling is conversion of a continuous signal into discrete-time signal obtained by taking samples. (Input: $x_a(t) \longleftrightarrow$ Output: $x_a(nT) \equiv x(n)$)

   $x_a(t) = Acos(2\pi Ft + \theta)$, where:

   - $-\infty < t < \infty$,
   - A is amplitude,
   - F is frequency (in cycles per second or Hertz, $Hz = \frac{1}{s}$), $-\infty < F < \infty$,
   - θ is phase in radians,
   - expression $2\pi F$ can be written shortly as Ω (and it is frequency in radians per seconds), $-\infty < \Omega < \infty$.

   $\Longrightarrow x_a(nT) \equiv x(n) = Acos(2\pi FnT + \theta) = Acos(\frac{2\pi nF}{F_S} + \theta) = Acos(2\pi nf + \theta)$, where:

   - $n \in \mathbb{Z}$,
   - T is sampling period in seconds,
   - F_s is sampling frequency in samples per second, $F_s = \frac{1}{T}$,
   - f is frequency in cycles per sample, $f = \frac{F}{Fs}$, $-\frac{1}{2} \leq f \leq \frac{1}{2}$,
   - expression $2\pi f$ can be written shortly as ω (and it is frequency in radians per sample), $-\pi \leq \omega \leq \pi$.

   Tasks:

   (a) Derive the expression for the discrete signal $x(n)$ obtained by sampling $x_a(t) = Asin(2\pi Ft)$ with sampling frequency Fs.

   (b)
   - In Matlab generate three signals, x_1, x_2, x_3, with duration of 3 seconds and following properties: $F_S = 8000Hz$, $x_1(F_1 = 220Hz, A_1 = 3)$, $x_2(F_2 = 880Hz, A_2 = 0.7)$, $x_3 = x_1 + x_2$.
   - Compare signals by plotting them (x coordinate should represent seconds). Compare signals also by listening to them.

2. **Generating tones/melody**

   Tasks:

   (a) Generate following tones: C_1, C_2 and C_3. Frequency of different tones can be find on the following website `http://www.phy.mtu.edu/~suits/notefreqs.html`.

   (b) In Matlab generate a melody by your choice: start with an empty vector, each tone in a melody has its own frequency in Hz, its starting point and its duration.

   Notes of some Slovenian children songs (determine the length of tones by yourself):

   i. Kuža pazi: CCCCDDDDEEDDCCC

   ii. Marko skače: EGGGEGGGEEDDCCCDEGGEEDDCC

3. **Aliasing, Nyquist Theorem**

   The Nyquist sampling theorem provides a prescription for the nominal sampling interval required to avoid aliasing. It may be stated simply as: The sampling frequency should be at least twice the highest frequency contained in the signal.

   $F_S \geq 2F_{max}$, where F_S is sampling frequency and F_{max} is the maximum frequency contained in the signal. If $F_S = F_{max}$, then F_S is called Nyquist frequency.

   Tasks:

   (a)
   - Consider analog signal $x_a(t) = 3cos(2\pi 50t)$. Determine the minimum frequency required to avoid aliasing.
   - Suppose that the signal is sampled at rate $F_s = 200Hz$. What is the discrete-time signal obtained after sampling?

   (b) Consider analog signal $x_a(t) = cos(50\pi t) + 10cos(300\pi t)$. Determine the Nyquist frequency for this signal.

4. **Representation of discrete-time system**

   Discrete-time (DT) system is an algorithm. According to the rules, from the input discrete-time signal $x(n)$, it produces the output, which is another discrete-time signal $y(n)$.

   Tasks:

   (a)
   - Draw a block diagram of discrete-time system, described by the following difference equation: $y(n) = \frac{1}{3}x(n) + \frac{1}{3}x(n-1) + \frac{1}{3}x(n-2)$.
   - Determine output signal $y(n)$ for the system described above using input signal $x(n) = \delta(n)$ (unit sample). Check result using Matlab function *impz*.

   (b) Draw a block diagram of discrete-time system, described by the following difference equation: $y(n) = \frac{1}{3}x(n) + x(n-3) - x(n+2)$.