Digital filters

- Filters
- DeciBels and cutoff frequency
- Simple IIR lowpass filter
- Simple IIR highpass filter
- Simple IIR bandpass filter
- Simple IIR bandstop filter
- Second order IIR filter
- Linear-phase filters
- Linear-phase FIR filters
- (Additional materials)

Filters

- Aim

  To separate information in frequency domain by proper construction of frequency response of LTI system. i.e., proper construction of transfer function of LTI system (amplitude and phase characteristic)

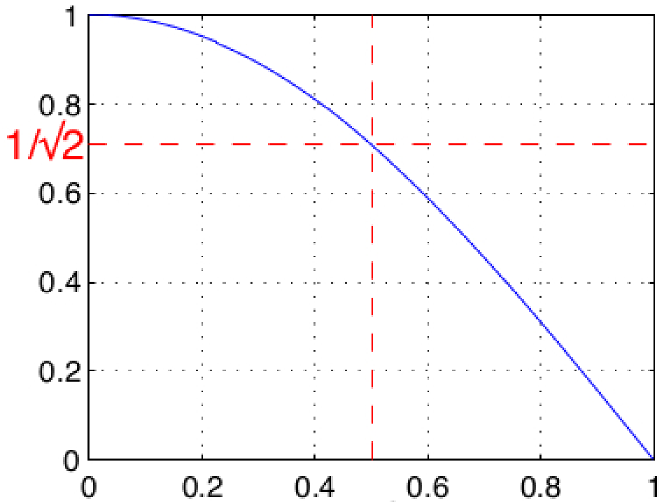
- What is filter?

  - A system that is altering signal (its frequency content) in an useful way

DeciBels and cutoff frequency

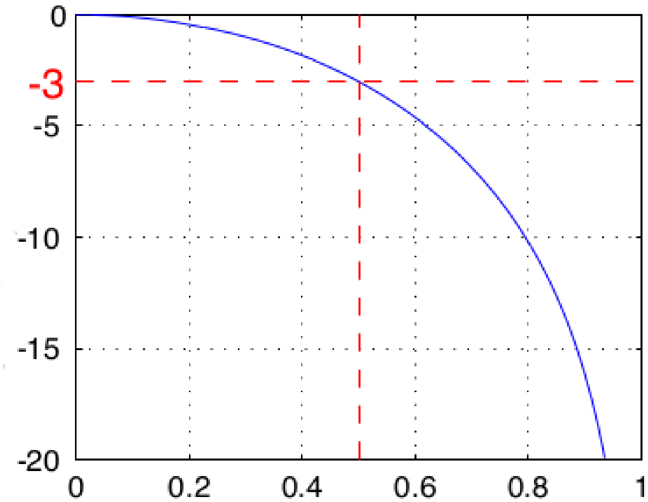
- Filter's amplitude response in linear scale and in dB

$\left|H(e^{j\omega})\right|$



$\frac{\omega}{\pi} = \frac{2\pi f}{\pi}$

$\left|H(e^{j\omega})\right|$ in dB



$\frac{\omega}{\pi}$

- A level of 0 corresponds to $-\infty$ dB

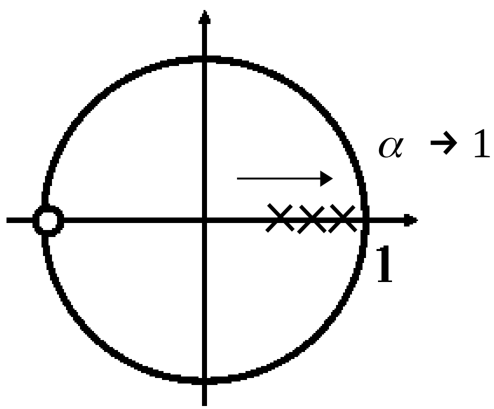
Simple IIR lowpass filter

- First-order IIR lowpass filter

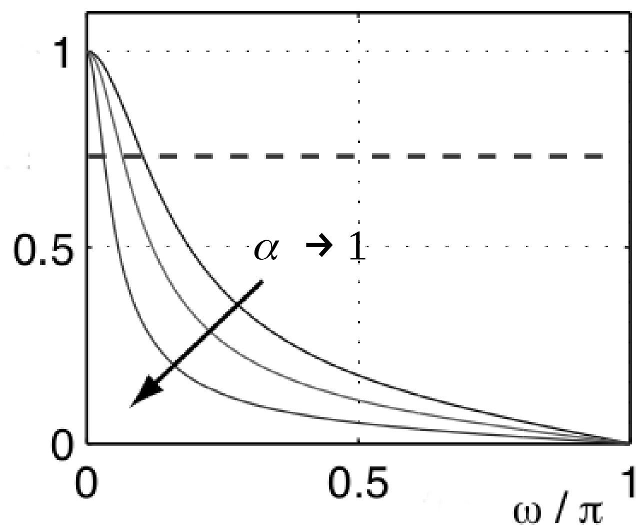
$$H_L(z) = K\frac{1+z^{-1}}{1-\alpha z^{-1}} = K\frac{z+1}{z-\alpha} \qquad y(n) = \alpha\, y(n-1) + K\,(x(n) + x(n-1))$$

K – scaling factor to make gain of 1 at $\omega = 0$, $H_L(1) = 1$, → $K = (1-\alpha)/2$

Zero, $z = -1$, pole $z = \alpha$



$|H_L(e^{j\omega})|$



Simple IIR lowpass filter

- First-order IIR lowpass filter

$$H_L(z) = K\frac{1+z^{-1}}{1-\alpha z^{-1}} = K\frac{z+1}{z-\alpha} \qquad K = \frac{(1-\alpha)}{2}$$

- Cutoff frequency ω_C from $\left|H_L(e^{j\omega})\right|^2 = \frac{1}{2}$

$$\left|H_L(e^{j\omega})\right|^2 = \frac{(1-\alpha)^2}{4}\frac{(1+e^{-j\omega_C})(1+e^{j\omega_C})}{(1-\alpha e^{-j\omega_C})(1-\alpha e^{j\omega_C})} = \frac{1}{2}$$

- Design equation $\Rightarrow \cos\omega_C = \frac{2\alpha}{1+\alpha^2} \qquad \alpha = \frac{1-\sin\omega_C}{\cos\omega_C}$

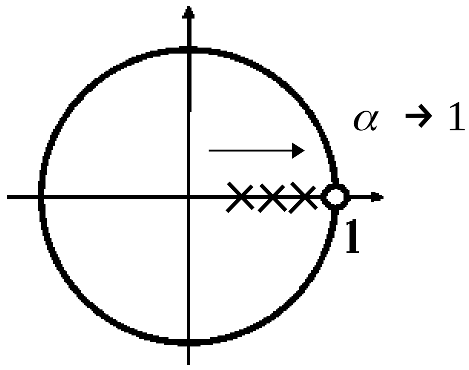
Simple IIR highpass filter

- First-order IIR highpass filter

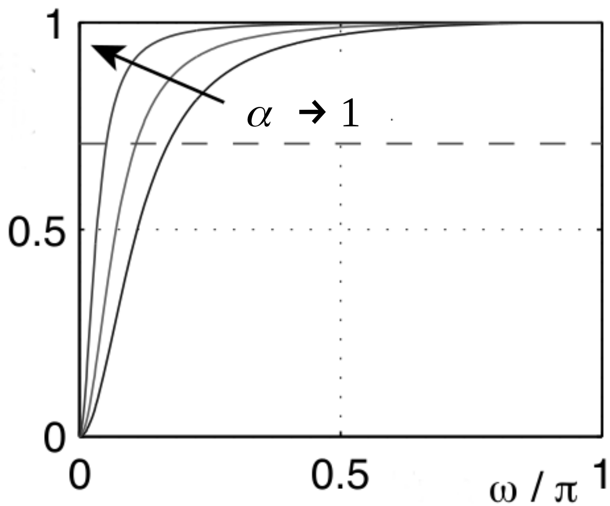
$$H_H(z) = K\frac{1-z^{-1}}{1-\alpha z^{-1}} = K\frac{z-1}{z-\alpha} \qquad y(n) = \alpha\, y(n-1) + K\,(x(n) - x(n-1))$$

K – scaling factor to make gain of 1 at $\omega = \pi$, $H_H(-1) = 1$, → $K = (1+\alpha)/2$

Zero, $z = 1$, pole $z = \alpha$



$|H_H(e^{j\omega})|$



Simple IIR highpass filter

- First-order IIR highpass filter

$$H_H(z) = K\frac{1 - z^{-1}}{1 - \alpha z^{-1}} = K\frac{z - 1}{z - \alpha} \qquad K = \frac{(1+\alpha)}{2}$$

- Cutoff frequency ω_C from $\left|H_H(e^{j\omega})\right|^2 = \frac{1}{2}$

$$\left|H_H(e^{j\omega})\right|^2 = \frac{(1+\alpha)^2}{4}\frac{(1-e^{-j\omega_C})(1-e^{j\omega_C})}{(1-\alpha e^{-j\omega_C})(1-\alpha e^{j\omega_C})} = \frac{1}{2}$$

- Design equation (again) $\Rightarrow \alpha = \dfrac{1-\sin\omega_C}{\cos\omega_C}$

Simple IIR bandpass filter

- Second-order IIR bandpass filter,

$$H_{BP}(z) = K \frac{(1+z^{-1})(1-z^{-1})}{1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}} \qquad y(n) = ?$$

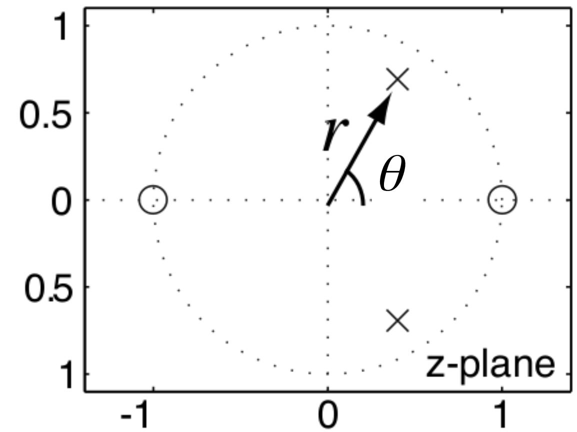
$$H_{BP}(z) = \frac{1-\alpha}{2} \frac{1-z^{-2}}{1 - \beta(1+\alpha)z^{-1} + \alpha z^{-2}}$$

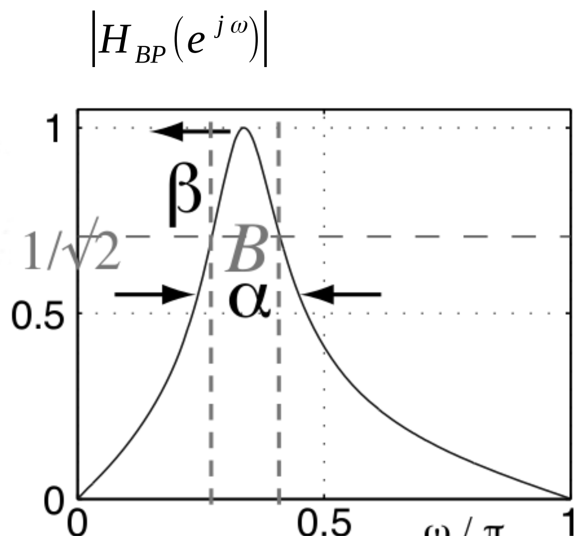
$$\text{where} \quad r = \sqrt{\alpha}, \quad \cos\theta = \frac{\beta(1+\alpha)}{2\sqrt{\alpha}}$$

- Design equations

ω_C is center frequency (not cutoff): $\beta = \cos\omega_C$

B is 3dB bandwidth: $\frac{2\alpha}{1+\alpha^2} = \cos B$





Simple IIR bandpass filter

- Design a second-order IIR bandpass filter with center frequency, $\omega_C = 0.4\pi$, and 3dB bandwidth, $B = 0.1\pi$

$$\omega_C = 0.4\pi \Rightarrow \beta = \cos\omega_C = 0.30901$$

$$B = 0.1\pi \Rightarrow \frac{2\alpha}{1+\alpha^2} = \cos(0.1\pi)$$

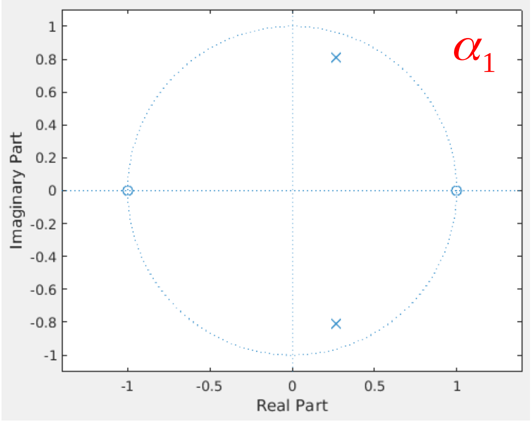
$$\Rightarrow \alpha_1 = 0.72654, \quad \alpha_2 = 1.37638 \quad ?$$

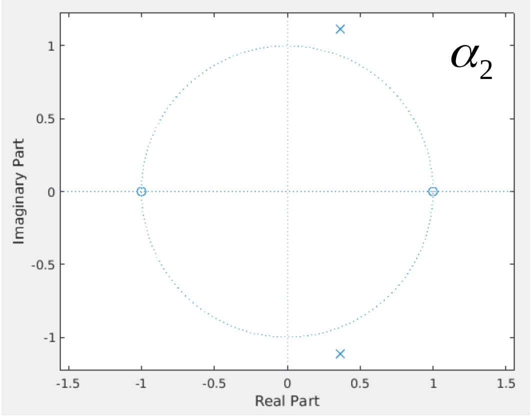
$$H_{BP}(z) = \frac{1-\alpha}{2}\,\frac{1-z^{-2}}{1-\beta(1+\alpha)^{-1}+\alpha z^{-2}}$$

$$H_{BP}(z) = 0.13673\,\frac{1-z^{-2}}{1-0.53353\,z^{-1}+0.72654\,z^{-2}}$$

Design equations: $\beta = \cos\omega_C$

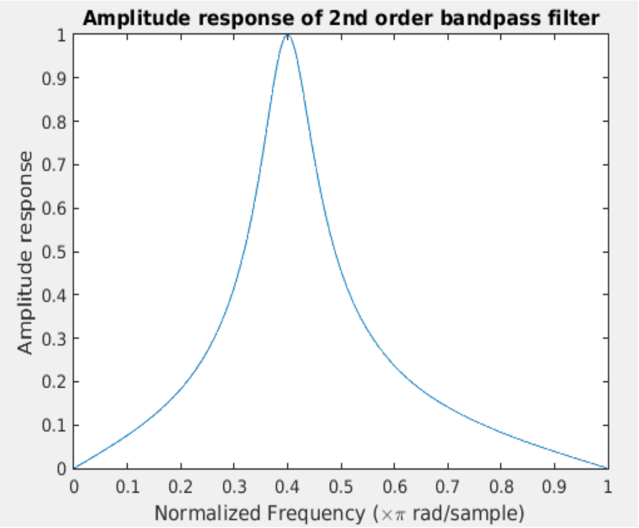
$$\frac{2\alpha}{1+\alpha^2} = \cos B$$

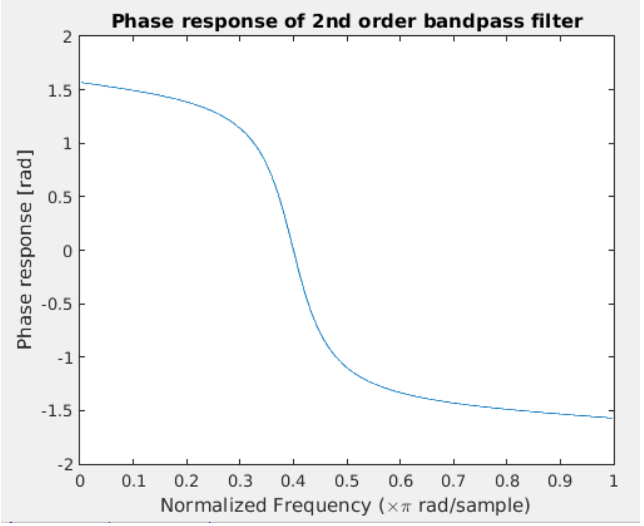




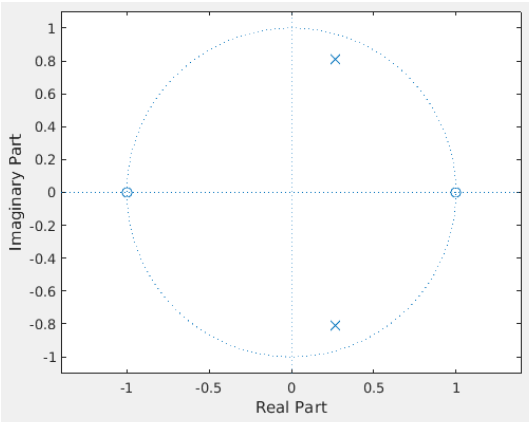
Simple IIR bandpass filter

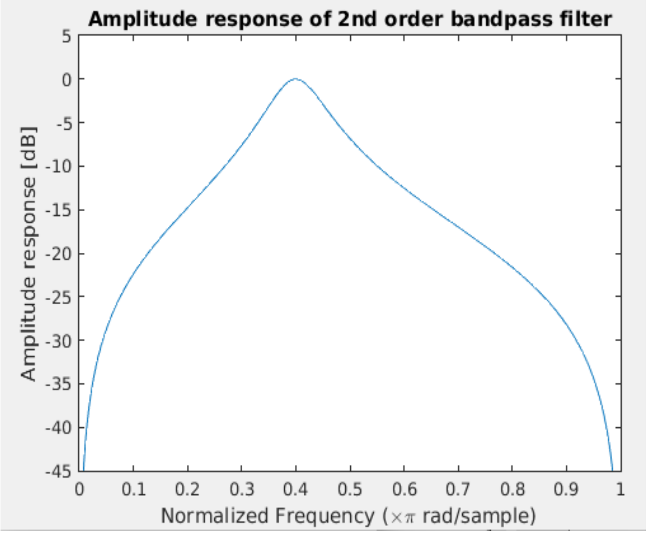
- Second-order IIR bandpass filter with center frequency, $\omega_C = 0.4\pi$, and 3dB bandwidth, $B = 0.1\pi$

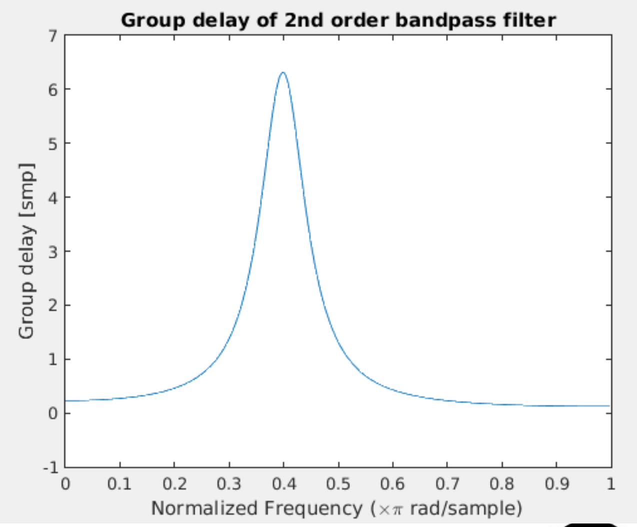




α_1







Simple IIR bandstop filter

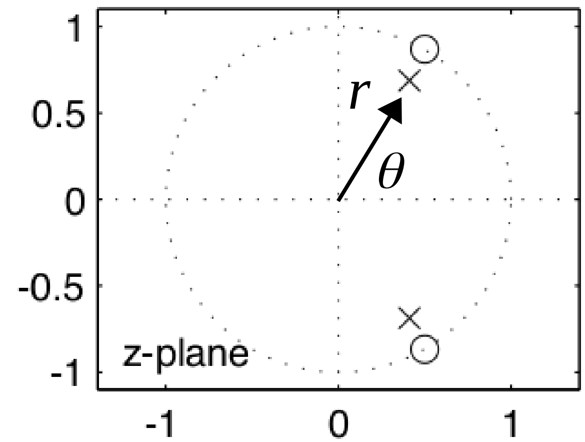
- Second-order IIR bandstop filter

$$H_{BS}(z) = K \frac{1 - 2\cos\theta\, z^{-1} + z^{-2}}{1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}} \qquad y(n) = ?$$

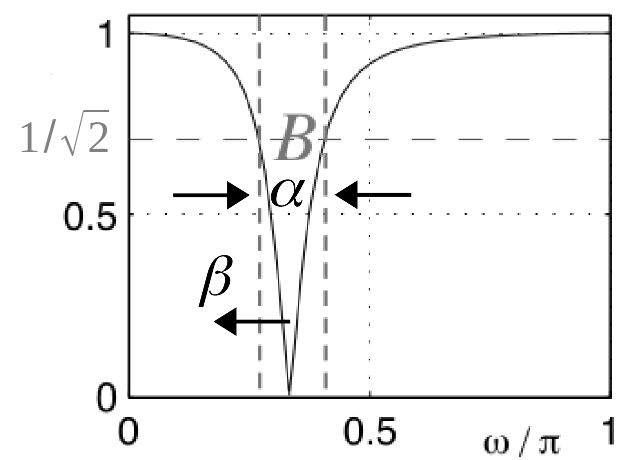
$$H_{BS}(z) = \frac{1+\alpha}{2} \frac{1 - 2\beta z^{-1} + z^{-2}}{1 - \beta(1+\alpha) z^{-1} + \alpha z^{-2}}$$

- zeros at ω_C, poles the same as for H_{BP}

$$\text{where } r = \sqrt{\alpha}, \quad \cos\theta = \frac{\beta(1+\alpha)}{2\sqrt{\alpha}}$$



$|H_{BS}(e^{j\omega})|$



- Design equations

ω_C is center frequency (not cutoff): $\beta = \cos\omega_C$

B is 3dB bandwidth: $\frac{2\alpha}{1+\alpha^2} = \cos B$

Second order IIR filter

- Homework, visit the following site

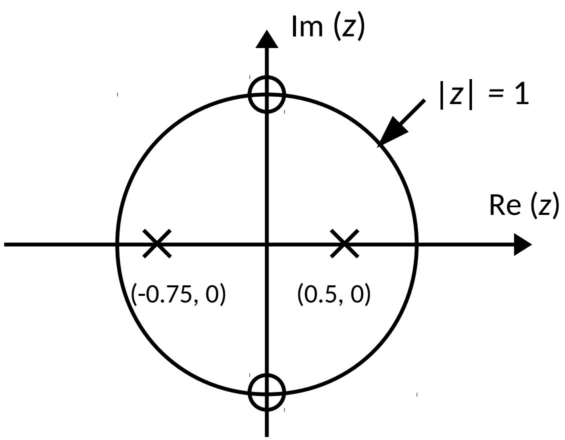
  http://www.earlevel.com/main/2013/10/28/pole-zero-placement-v2/

  - Verify amplitude response of the second order IIR filter by manipulating the positions of zeros and poles in the Z plane
  - Write the corresponding transfer functions and difference equations for the selected positions
  - Comment on causality
  - In what case the filter becomes an FIR filter?

Second order IIR filter

- Example of exam task

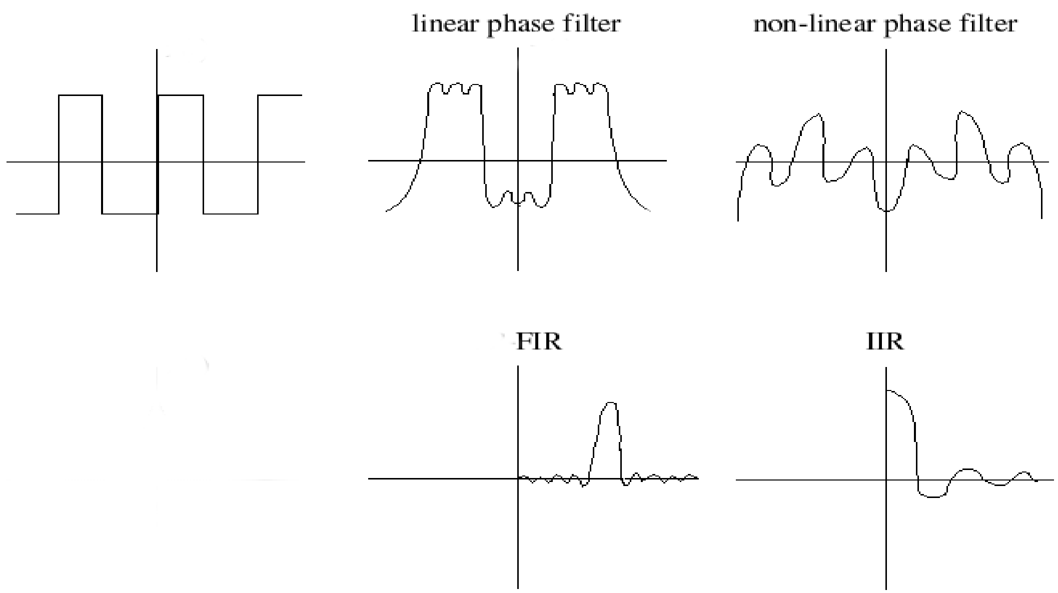
A zero-pole diagram, given H(z) of discrete linear time-invariant system is following:



On the basis of the zero-pole diagram write the transfer function H(z) of this system. Is this system stable? You have to justify your answer. Sketch also the amplitude response of this system.

Linear-phase filters

- Causal FIR filters (real impulse response, *h(n)*) can have linear phase
  - *Impulse response, h(n), is of finite duration → can be symmetric*
  - *If impulse response, h(n), is symmetric → linear phase*
- Causal IIR filters (real impulse response, *h(n)*) cannot have linear phase
  - *Impulse response, h(n), is of infinite duration → cannot be symmetric*
  - *Since impulse response, h(n), is not symmetric → non-linear phase*



Linear-phase FIR filters

- It is possible to design an FIR filter with exact linear phase response
- A causal FIR transfer function *H(z)* of length *M+1*, order *M*,

$$H(z) = \sum_{n=0}^{M} h(n) z^{-n}$$

has a liner phase, if its impulse response, *h(n)*, is symmetric,

$$h(n) = h(M-n), \quad 0 \le n \le M$$

or antisymmetric

$$h(n) = -h(M-n), \quad 0 \le n \le M$$

- *For an FIR filter with a real impulse response, the zeros of H(z) occur in complex conjugate pairs*
- Since the length can be even or odd, yields four types of linear phase FIR transfer functions

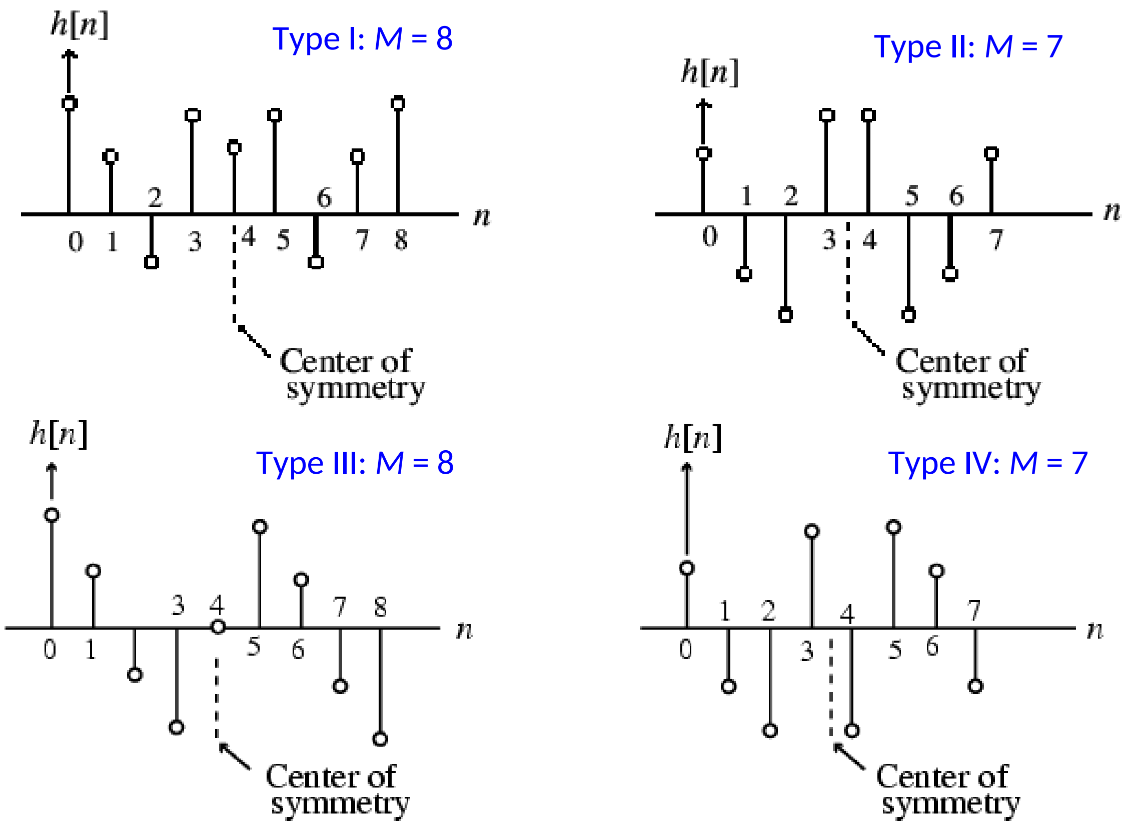
Linear-phase FIR filters

- Symmetric and anti-symmetric FIR filters are almost the only one to get linear phase

Odd length (M+1) | Even length (M+1)

Symmetric

Anti-symmetric



Linear-phase FIR filters

- Example of exam task

An impulse response of a digital filter is following: $h(n) = \{1, 0, -1\}$.

Is this filter with finite or infinite impulse response? Is this filter with linear or non-linear phase? What is the order of this filter? Derive transfer function, frequency response, amplitude response, phase response, and group delay of this filter. Sketch zero-pole diagram, amplitude response, phase response, and group delay.

(Additional materials)

- MATLAB and filter visualization and design tool (fvtool, sptool, fdatool)
- Classification of filters
- Simple FIR lowpass filter
- Simple FIR highpass filter
- Linear-phase FIR filters
- Linear-phase FIR filters, Type I, Type II, Type III, Type IV
- Zero locations of linear-phase FIR transfer functions

MATLAB and filter visualization and design tools (fvtool, sptool, fdatool)

- FVTOOL

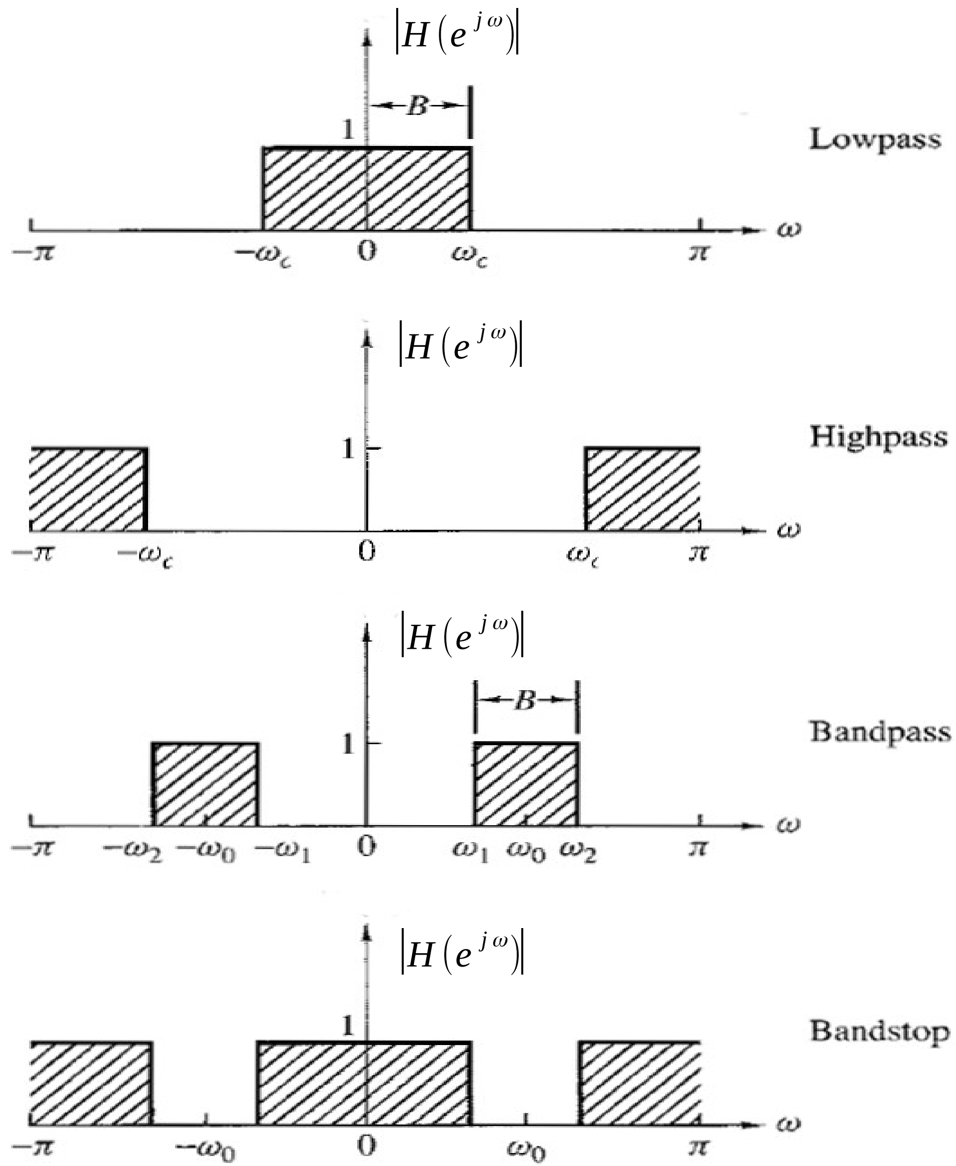
- Explore MATLAB's filter visualization and design tools

```
MATLAB
>> fvtool(b, a);        % Filter Visualization Tool
>> sptool               % Interactive digital signal processing tool
>>
>> fdatool              % Filter Design and Analysis Tool
>>                      % It design and analyze filters, and modify existing
>>                      % filter designs
>> ...
```

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

Classification of filters

- Usual classification
  - Lowpass
  - Highpass
  - Bandpass
  - Bandstop
- Desired
  - Amplitude responses with constant-gain passband characteristics and with zero gain in their stop bands
  - Phase responses linear

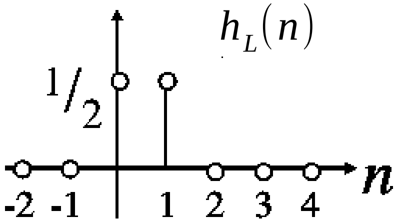


[Proakis, Manolakis]

Simple FIR lowpass filter

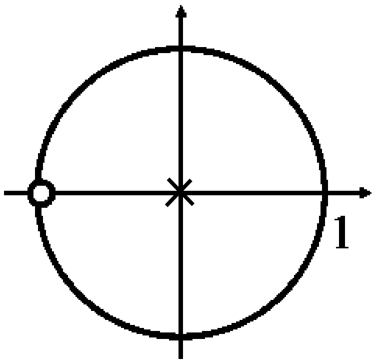
- Moving average over two samples

$h_L(n) = \{1/2,\ 1/2\}$



$$H_L(z) = \frac{1}{2}(1+z^{-1}) = \frac{z+1}{2z} \qquad y(n) = \frac{1}{2}(x(n) + x(n-1))$$

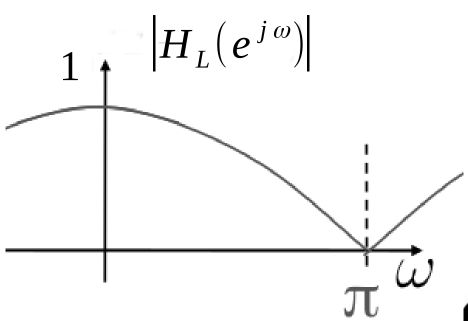
$$H_L(z) = \frac{1}{2} z^{-1/2}(z^{1/2}+z^{-1/2})$$



$$H_L(e^{j\omega}) = \frac{1}{2} e^{-j\omega/2}(e^{j\omega/2}+e^{-j\omega/2})$$

$$H_L(e^{j\omega}) = e^{-j\omega/2}\cos(\omega/2)$$

$$e^{j\omega/2} + e^{-j\omega/2}$$



- Delay of 1/2 sample
- Zero at $z = -1$ (Pole at $z = 0$)

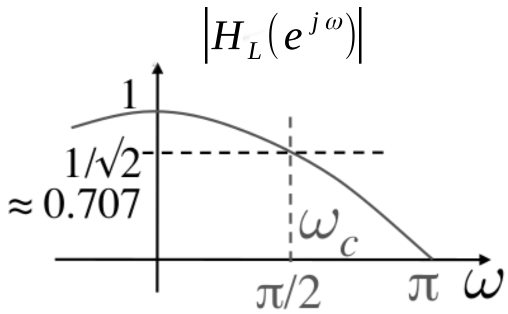
Simple FIR lowpass filter

- Moving average over two samples

$h_L(n) = \{1/2,\ 1/2\}$

$$H_L(z) = \frac{1}{2}\left(1+z^{-1}\right) = \frac{z+1}{2z}$$

$$H_L(e^{j\omega}) = e^{-j\omega/2}\cos(\omega/2)$$



- Filters are characterized by cutoff frequency, ω_C

defined as one half, 1/2, power point, or -3dB point

$$\left|H(e^{j\omega_C})\right|^2 = \frac{1}{2}\max\left\{\left|H(e^{j\omega})\right|^2\right\} \Rightarrow |H| = \frac{1}{\sqrt{2}}H_{max}$$

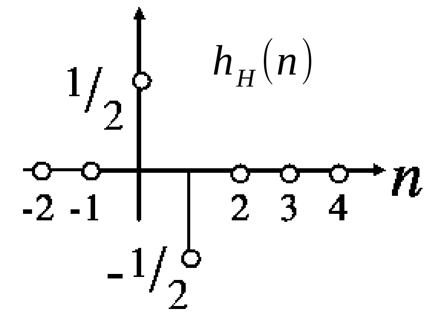
$$\left|H_L(e^{j\omega})\right| = \cos(\omega/2)$$

$$\omega_C = 2\arccos\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{2}$$

Simple FIR highpass filter

- Differentiating over two samples

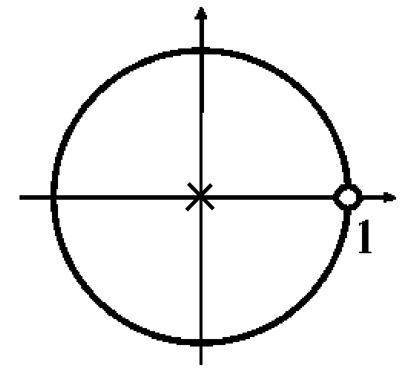
$h_H(n) = \{1/2,\ -1/2\}$



$$H_H(z) = \frac{1}{2}(1-z^{-1}) = \frac{z-1}{2z} \qquad y(n) = \frac{1}{2}(x(n) - x(n-1))$$

$$H_H(z) = \frac{1}{2} z^{-1/2}(z^{1/2} - z^{-1/2})$$

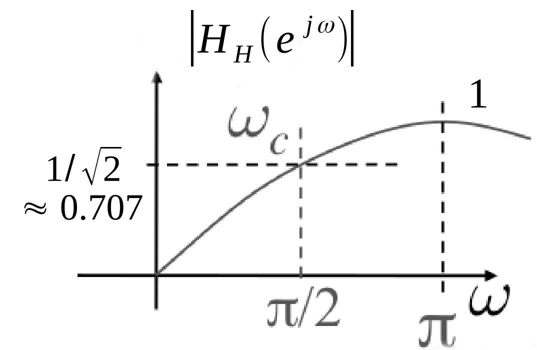
$$H_H(e^{j\omega}) = j\, e^{-j\omega/2} \sin(\omega/2)$$



- Zero at $z = 1$ (Pole at $z = 0$)
- One half, 1/2, sample delay
- Cutoff frequency ω_C

$$|H| = \frac{1}{\sqrt{2}} H_{max} \qquad |H_H(e^{j\omega})| = |\sin(\omega/2)|$$

$$\omega_C = 2 \arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{2} \qquad \text{(again)}$$



Linear-phase FIR filters

- Visit the following site

  https://www.youtube.com/watch?v=KVOkWcknvc4

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Linear-phase FIR filters, Type I

- Length *M*+1 (odd), order *M* (even)
- Symmetric $h(n) = h(M-n)$

$$H(e^{j\omega}) = \sum_{n=0}^{M} h(n) e^{-j\omega n} = e^{-jM\omega/2}\,\widetilde{H}(\omega)$$

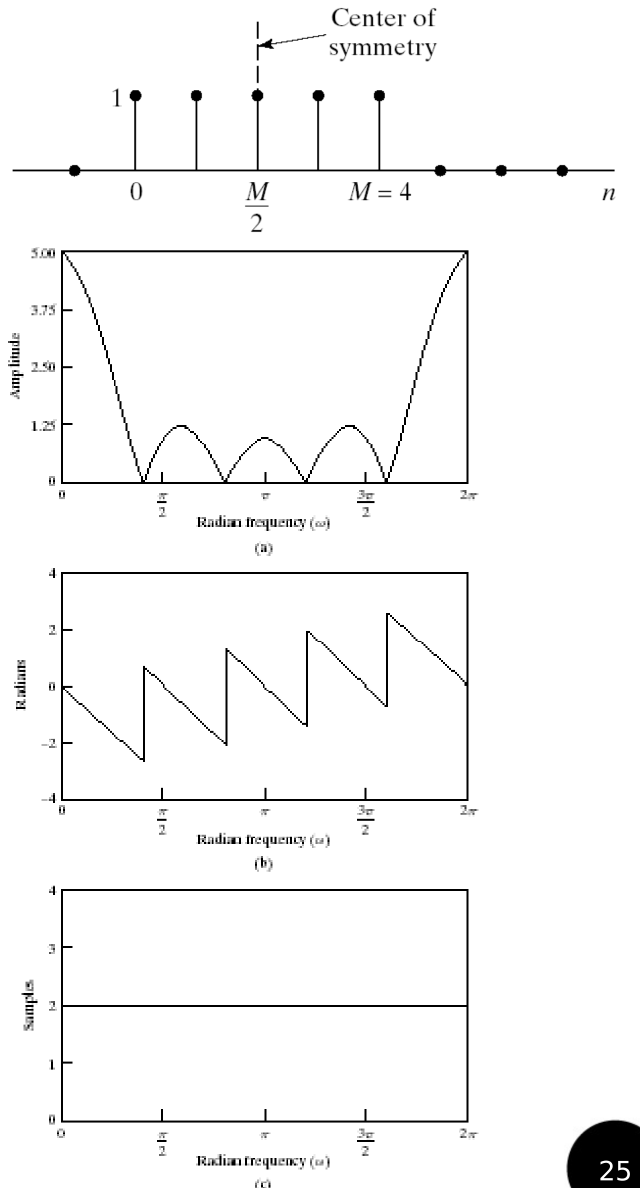
$$\widetilde{H}(\omega) = h\left(\frac{M}{2}\right) + 2\sum_{n=1}^{M/2} h\left(\frac{M}{2}-n\right)\cos(\omega n)$$

- Pure real (from cosine basis)
- Phase response

$$\theta(\omega) = -\omega\frac{M}{2}$$

- Group delay is constant

$$\tau_G(\omega) = \frac{-d\,\theta(\omega)}{d\omega} = \frac{M}{2}$$



Linear-phase FIR filters, Type II

- Length M+1 (even), order M (odd)
- Symmetric $h(n) = h(M-n)$

$$H(e^{j\omega}) = \sum_{n=0}^{M} h(n) e^{-j\omega n} = e^{-jM\omega/2}\,\widetilde{H}(\omega)$$

$$\widetilde{H}(\omega) = 2\sum_{n=1}^{(M+1)/2} h\left(\frac{M+1}{2}-n\right)\cos\left(\omega\left(n-\frac{1}{2}\right)\right)$$

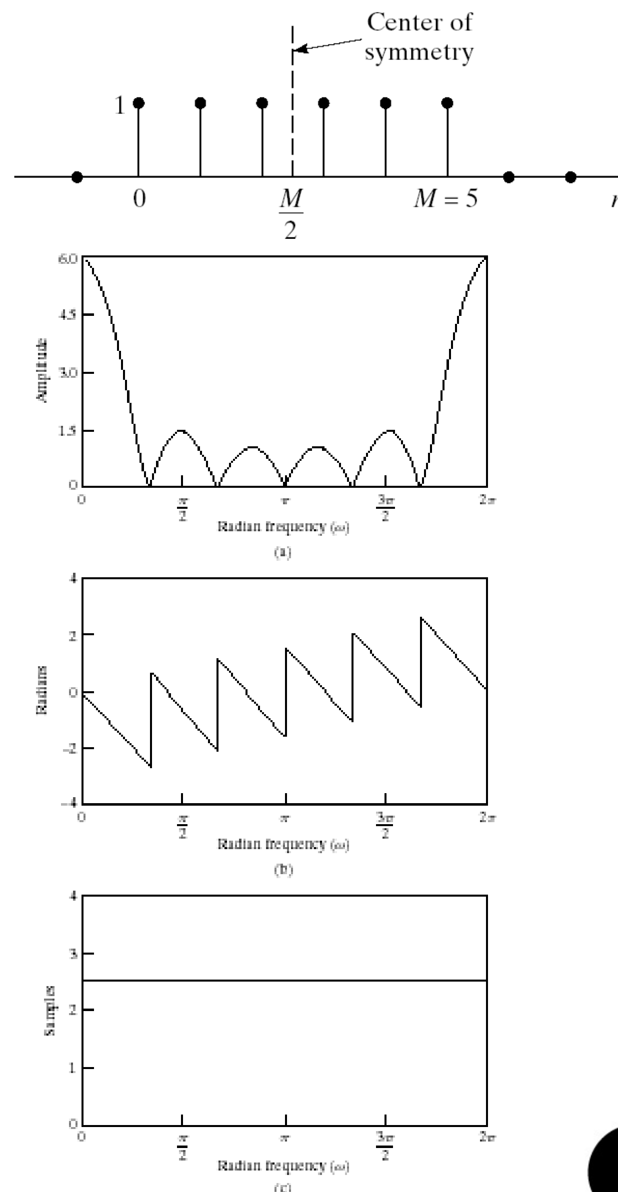
- Pure real (from cosine basis)
  (Always zero at) $\omega = \pi$
- Phase response

$$\theta(\omega) = -\omega\frac{M}{2}$$

- Group delay is constant (non-integer delay)

$$\tau_G(\omega) = \frac{-d\theta(\omega)}{d\omega} = \frac{M}{2}$$

Homework:
Verify that for M=1
→ Simple FIR lowpass



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Linear-phase FIR filters, Type III

- Length *M*+1 (odd), order *M* (even)
- Anti-symmetric $h(n) = -h(M-n)$

$$H(e^{j\omega}) = \sum_{n=0}^{M} h(n) e^{-j\omega n} = j e^{-jM\omega/2} \tilde{H}(\omega)$$

$$\tilde{H}(\omega) = 2\sum_{n=1}^{M/2} h\left(\frac{M}{2}-n\right)\sin(\omega n)$$

- Pure real (from sine basis)

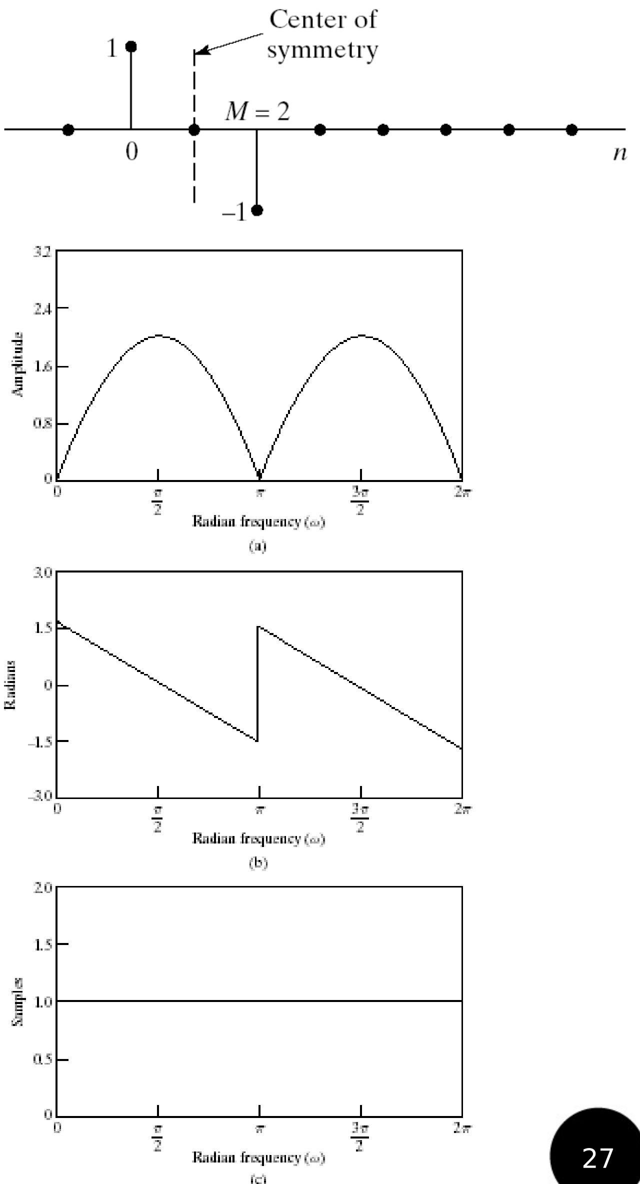
  (Always zero at) $\omega = 0$ $\omega = \pi$

- Phase response (additional shift) $\pi/2$

$$\theta(\omega) = -\omega\frac{M}{2} + \frac{\pi}{2}$$

- Group delay is constant

$$\tau_G(\omega) = \frac{-d\theta(\omega)}{d\omega} = \frac{M}{2}$$



Linear-phase FIR filters, Type III

- Example, Type III   *h(n)* = {1, 1, 0, -1, -1} = {*h(0)*, *h(1)*, *h(2)*, *h(3)*, *h(4)*}
  = {*h(0)*, *h(1)*, 0, -*h(1)*, -*h(0)*}
- Anti-symmetric $h(n) = -h(M-n), \quad M = 4$

$$H(e^{j\omega}) = \sum_{n=0}^{M} h(n) e^{-j\omega n} = e^{j\theta(\omega)} \widetilde{H}(\omega) \qquad \widetilde{H}(\omega) \rightarrow \sum (\sin(\omega n))$$

$$= h(0) + h(1) e^{-j\omega} - h(1) e^{-j3\omega} - h(0) e^{-j4\omega}$$

$$= e^{-j2\omega} \left(h(0)(e^{j2\omega} - e^{-j2\omega}) + h(1)(e^{j\omega} - e^{-j\omega})\right)$$

$$= e^{-j2\omega}\, j\, 2\, (h(0)\sin(2\omega) + h(1)\sin(\omega))$$

$$H(e^{j\omega}) = e^{-j2\omega} e^{j\frac{\pi}{2}}\, 2\, (h(0)\sin(2\omega) + h(1)\sin(\omega))$$

$$\widetilde{H}(\omega) = 2\sum_{n=1}^{2} h(4-n)\sin(\omega n) \qquad \theta(\omega) = -2\omega + \frac{\pi}{2} \qquad \tau_G(\omega) = \frac{-d\,\theta(\omega)}{d\,\omega} = 2$$

Linear-phase FIR filters, Type III

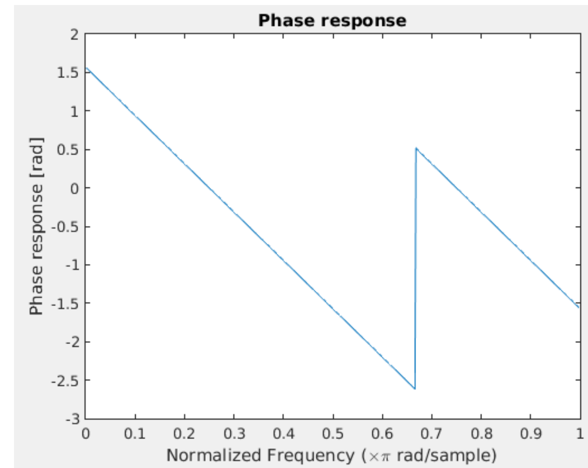
- Example, Type III   *h(n)* = {1, 1, 0, -1, -1} = {*h(0)*, *h(1)*, 0, -*h(1)*, -*h(0)*}

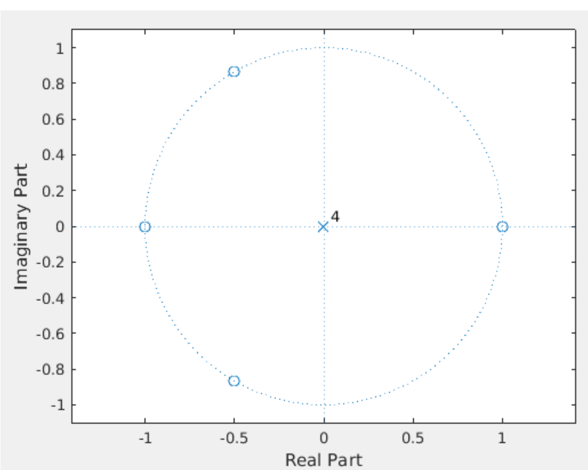
$$H(z) = 1 + z^{-1} - z^{-3} - z^{-4}$$

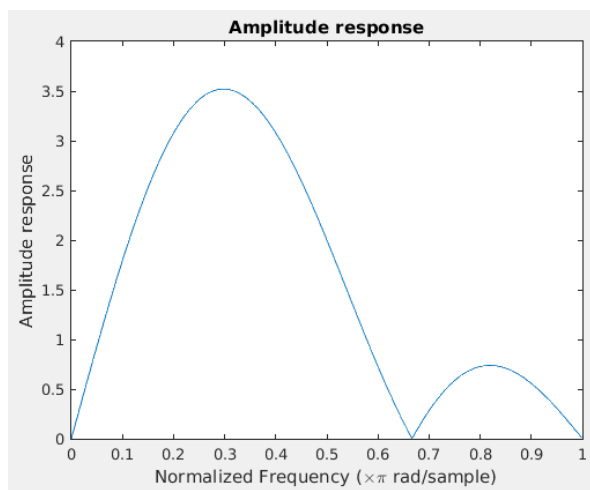
$$H(z) = \left(1 + z^{-1} + z^{-2}\right)\left(1 - z^{-2}\right)$$

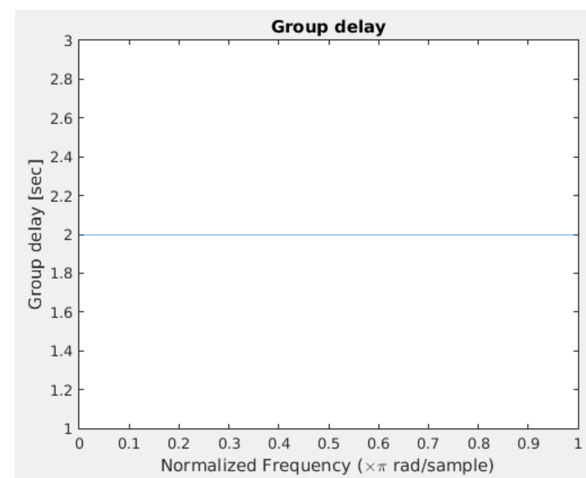
→ Moving average over 3 samples followed by
the first order differentiator

$$H(z) = \frac{\left(1 - z^{-3}\right)}{\left(1 - z^{-1}\right)}\left(1 - z^{-2}\right)$$









Linear-phase FIR filters, Type III

- Example of exam task

$h(n) = \{1, 0, -1\} = \{h(0), h(1), h(2)\}$
$= \{h(0), 0, -h(0)\}$

- Anti-symmetric (Type III)

$$h(n) = -h(M-n), \quad M = 2$$

$$H(e^{j\omega}) = \sum_{n=0}^{M} h(n) e^{-j\omega n} = e^{j\theta(\omega)} \left| H(e^{j\omega}) \right|$$

$$= h(0) + h(2) e^{-j2\omega} = h(0) - h(0) e^{-j2\omega} = 1 - e^{-j2\omega}$$

$$= e^{-j\omega} \left(e^{j\omega} - e^{-j\omega} \right)$$

$$= e^{-j\omega} \, j \, 2 \sin(\omega)$$

$$H(e^{j\omega}) = e^{-j\omega} e^{j\frac{\pi}{2}} \, 2\sin(\omega)$$

$$\theta(\omega) = -\omega + \frac{\pi}{2}$$

$$\left| H(e^{j\omega}) \right| = \widetilde{H}(\omega) = \left| 2 \sin(\omega) \right|$$

$$\tau_G(\omega) = \frac{-d\,\theta(\omega)}{d\,\omega} = 1$$

Linear-phase FIR filters, Type IV

- Length $M+1$ (even), order M (odd)
- Anti-symmetric $h(n) = -h(M-n)$

$$H(e^{j\omega}) = \sum_{n=0}^{M} h(n) e^{-j\omega n} = j e^{-jM\omega/2}\, \widetilde{H}(\omega)$$

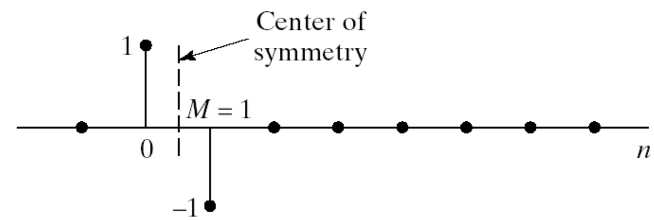
$$\widetilde{H}(\omega) = 2 \sum_{n=1}^{(M+1)/2} h\left(\frac{M+1}{2} - n\right) \sin\left(\omega\left(n - \frac{1}{2}\right)\right)$$

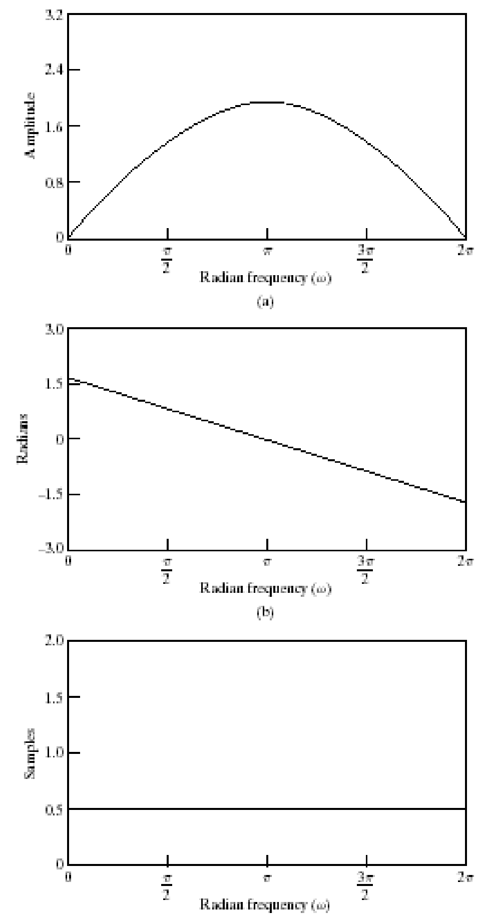
- Pure real (from sine basis)
- (Always zero at) $\omega = 0$
- Phase response (additional shift) $\pi/2$

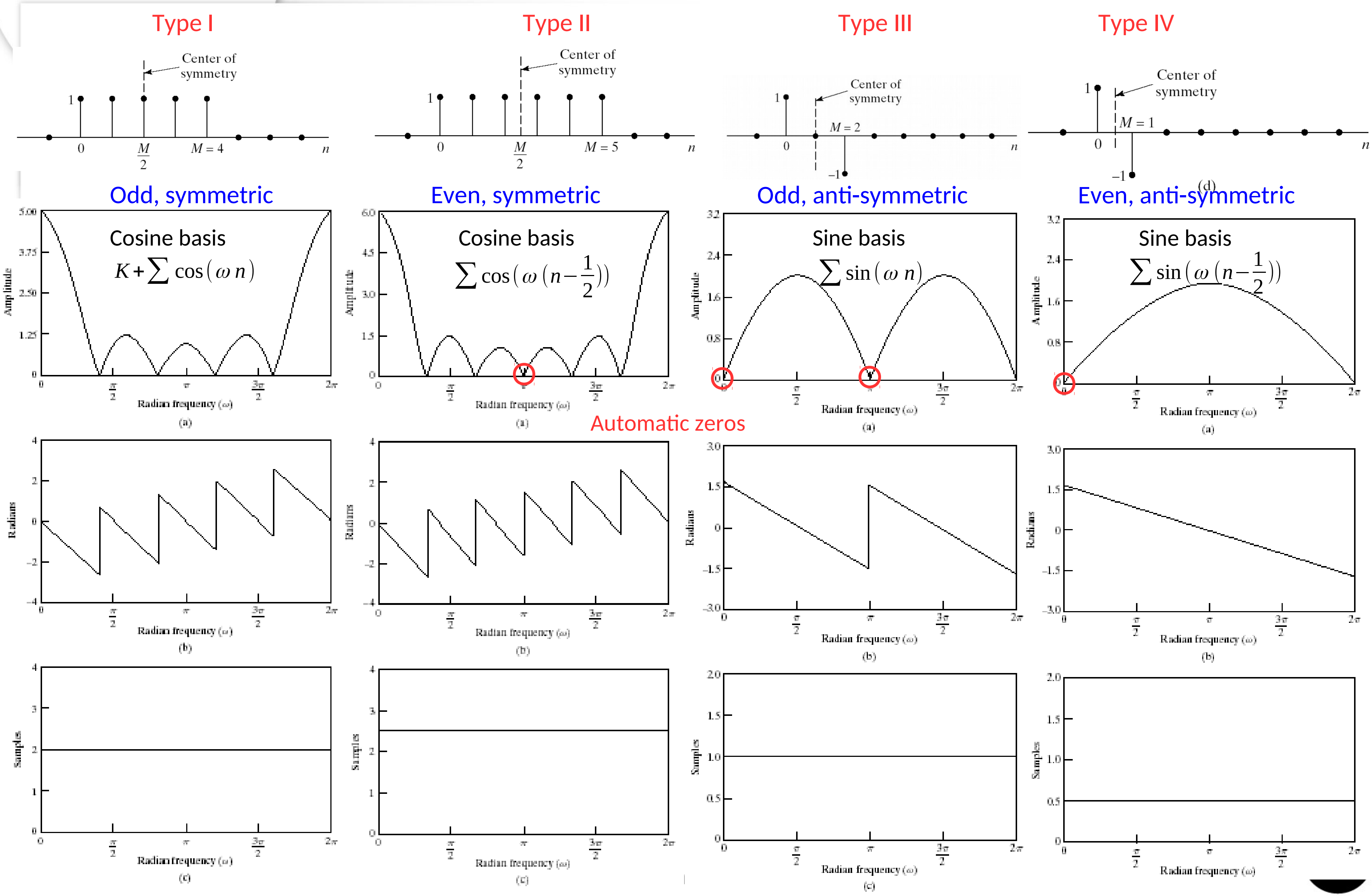
$$\theta(\omega) = -\omega \frac{M}{2} + \frac{\pi}{2}$$

- Group delay is constant (non-integer delay)

$$\tau_G(\omega) = \frac{-d\,\theta(\omega)}{d\omega} = \frac{M}{2}$$






Type I
Type II
Type III
Type IV
Center of symmetry
M = 4
M = 5
M = 2
M = 1
Odd, symmetric
Even, symmetric
Odd, anti-symmetric
Even, anti-symmetric
Cosine basis
$K+\sum\cos(\omega n)$
Cosine basis
$\sum\cos(\omega(n-\frac{1}{2}))$
Sine basis
$\sum\sin(\omega n)$
Sine basis
$\sum\sin(\omega(n-\frac{1}{2}))$
Automatic zeros
Amplitude
Radians
Samples
Radian frequency (ω)
(a)
(b)
(c)
(d)

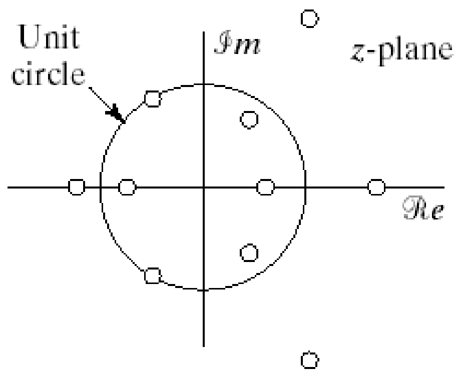
Linear-phase FIR filters

- Typical zero locations

Can be highpass, lowpass, bandstop, bandpass

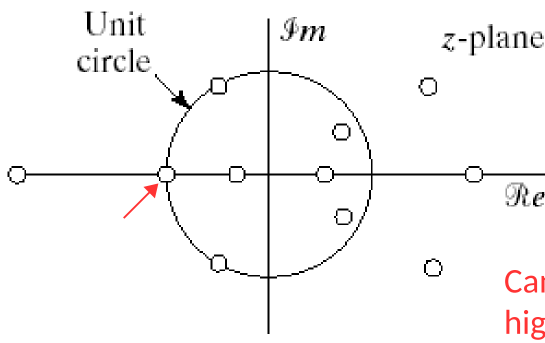
Type	automatic zeros
I	—
II	$\omega = \pi$
III	$\omega = 0, \pi$
IV	$\omega = 0$

Type I



(a)

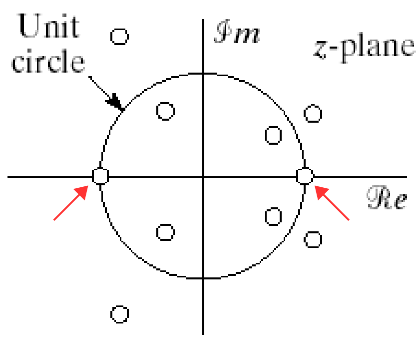
Type II



(b)

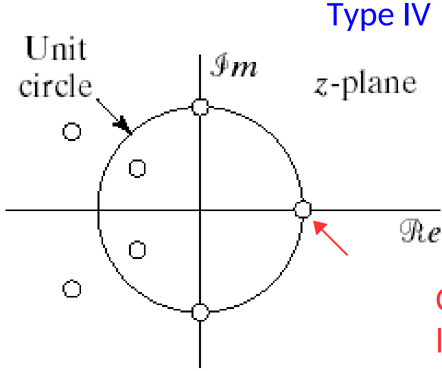
Cannot be highpass

Type III



Cannot be highpass, lowpass, bandstop

Type IV



Cannot be lowpass

Linear-phase FIR filters

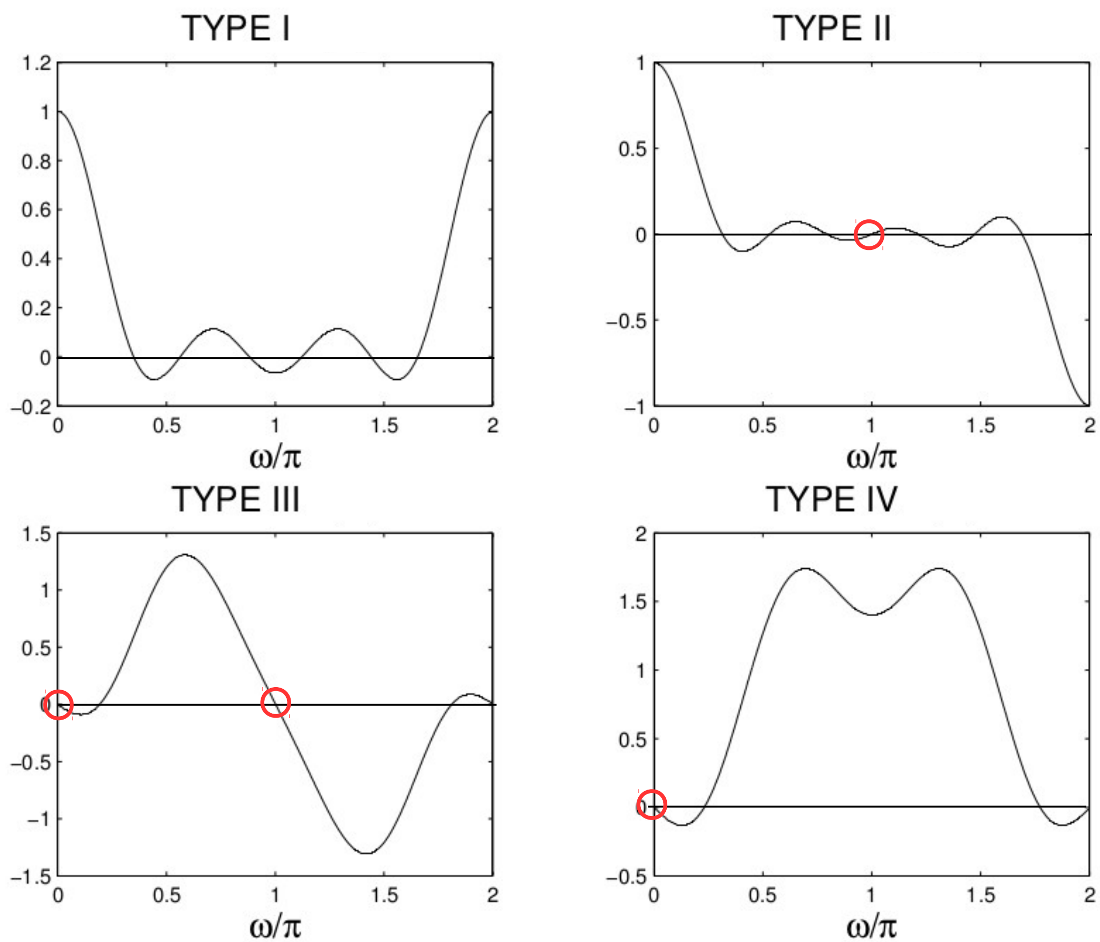
- Typical frequency responses, $\widetilde{H}(\omega)$ $\qquad H(e^{j\omega}) = e^{-jM\omega/2}\,\widetilde{H}(\omega)$

$K + \sum \cos(\omega\, n)$

$\sum \cos(\omega\,(n - \frac{1}{2}))$

$\sum \sin(\omega\, n)$

$\sum \sin(\omega\,(n - \frac{1}{2}))$



Zero locations of linear-phase FIR transfer functions

- An FIR filter with a symmetric impulse response (Type I and II)

$$h(n) = h(M-n)$$

- Transfer function can be written as $$H(z) = \sum_{n=0}^{M} h(n) z^{-n} = \sum_{n=0}^{M} h(M-n) z^{-n}$$
- If using $$m = M - n$$

$$\sum_{n=0}^{M} h(M-n) z^{-n} = \sum_{m=0}^{M} h(m) z^{-M+m} = z^{-M} \sum_{m=0}^{M} h(m) z^{m}$$

since $$\sum_{m=0}^{M} h(m) z^{m} = H(z^{-1})$$

follows $$H(z) = z^{-M} H(z^{-1})$$

- A real-coefficient polynomial, *H(z)*, satisfying this condition is a mirror-image polynomial

For Type III and IV

$$H(z) = -z^{-M} H(z^{-1})$$

Antimirror-image polynomial

Zero locations of linear-phase FIR transfer functions

- An FIR filter with a anti-symmetric impulse response (Type III and IV)

$$h(n) = -h(M-n)$$

- Transfer function can be written as $H(z) = \sum_{n=0}^{M} h(n) z^{-n} = -\sum_{n=0}^{M} h(M-n) z^{-n}$
- If using $m = M - n$

$$-\sum_{n=0}^{M} h(M-n) z^{-n} = -\sum_{m=0}^{M} h(m) z^{-M+m} = -z^{-M} \sum_{m=0}^{M} h(m) z^{m}$$

since $\sum_{m=0}^{M} h(m) z^{m} = H(z^{-1})$

follows $H(z) = -z^{-M} H(z^{-1})$

- A real-coefficient polynomial, *H(z)*, satisfying this condition is a antimirror-image polynomial

Zero locations of linear-phase FIR transfer functions

- Type II

  (Degree M odd)

$$H(z) = z^{-M} H(z^{-1})$$

$$H(-1) = (-1)^{-M} H(-1) = -H(-1)$$

$$\Rightarrow H(-1) = 0 \quad \text{Must have a zero at } z = -1$$

- Type III and IV

$$H(z) = -z^{-M} H(z^{-1})$$

$$H(1) = -(1)^{-M} H(1) = -H(1)$$

$$\Rightarrow H(1) = 0 \quad \text{Must have a zero at } z = 1$$

- Type III

  (Degree M even)

$$H(-1) = -(-1)^{-M} H(-1) = -H(-1)$$

$$\Rightarrow H(-1) = 0 \quad \text{Must have a zero at } z = -1$$

Zero locations of linear-phase FIR transfer functions

- From the relations

Type I, II: $H(z) = z^{-M} H(z^{-1})$

Type III, IV: $H(z) = -z^{-M} H(z^{-1})$

- It follows
  - If $z = z_0$ is a zero, $z = 1/z_0$ is also a zero
  - Since h(n) is real and z_0 is a zero, z_0^* is also a zero
  - A complex zero that is not on the unit circle is associated with four zeros
  - A complex zero on the unit circle is associated with two zeros as its reciprocal is also complex conjugate
  - A zero on the real line is associated with two zeros
  - Zeros at 1 and -1 do not imply the existence of zeros at other specific points
- Examples